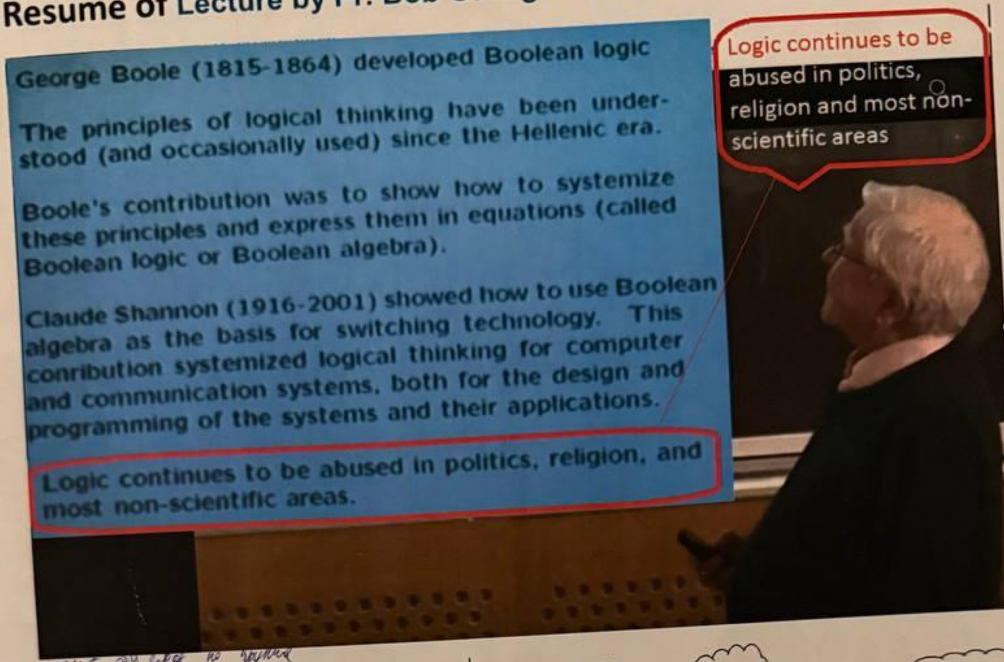
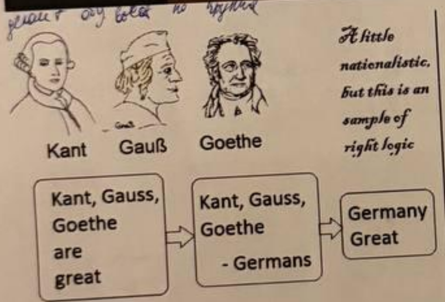
Resume of Lecture by Pr. Bob Gallagher from MIT

Massachusetts Institute of Technology (MIT)





Bad logic (abuse of logic)

Modeled communication as a system, invented the bit, focused on structure and statistics

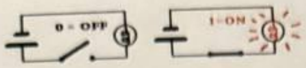
mathematical study of how information is quantified, stored and communicated

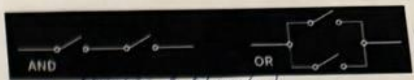
Algebraic manipulation of binary truth values

Lecture by Pr. Bob Gallagher
Boole (1815-1864) & **Shannon** (1916-2001)

Massachusetts Institute of Technology (MIT)

Sapere aude!





with (George Boole (19 century))

Logical addition (disjunction)

A	B	F=A∨B
0	0	0
0	1	1
1	0	1
1	1	1

Conjunction:		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

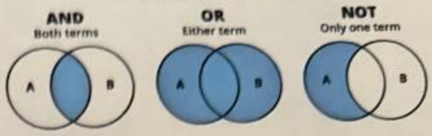
Logical disjunction

OR

Definition
Truth table
Logic gate

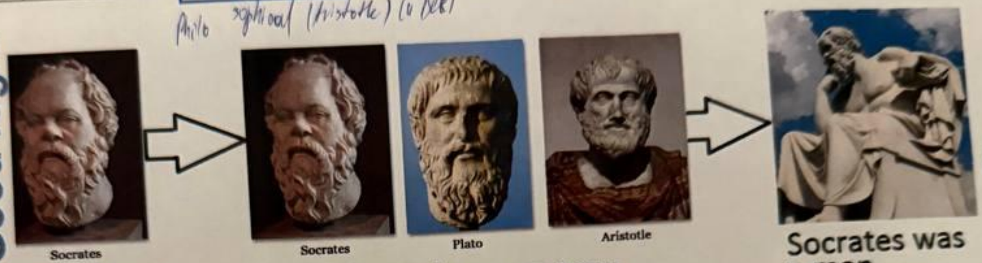
$p + q$
(0111)

BOOLEAN LOGIC



Philosophical (Aristotle) (4 BC)

Good logic



Socrates was a philosopher

$S \in \Phi$

philosophers are men

$\Phi \in A$

Socrates was a man

$S \in A$

More concerned with the structure of arguments and drawing conclusions from premises, often represented in natural language rather than abstract symbols

Bad logic

Socrates was a man

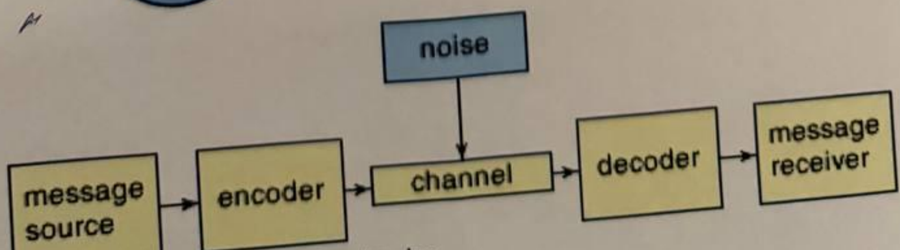
$S \in A$

philosophers are men

$\Phi \in A$

Socrates was a philosopher

$S \in \Phi$



XOR

0	0	0
0	1	1
1	0	1
1	1	

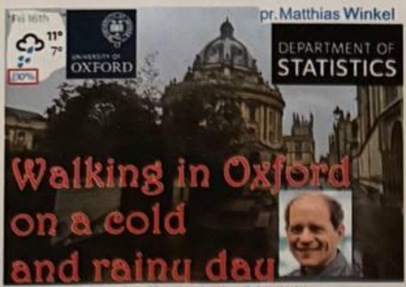
$0101_2 \to 5_{10}$

$01111_2 \to 15$

```
class MBoole
{
    public static void Main()
    {
    }
}
```

```
System.Console.WriteLine("Hi");
```

Solve as MBoole.cs



Markoff Chain Probability Model for Oxford Weather

CHALK+TALK ink+think

take notes on the lecture yourself

Motivation: 80% chance of rain

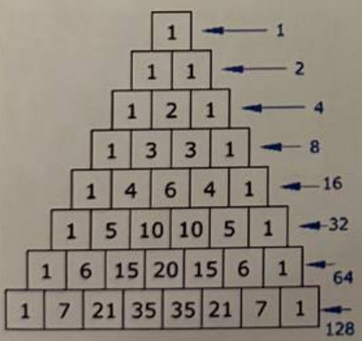
Let A_j be the event of rain at 9am on day j of this term, $1 \le j \le n$

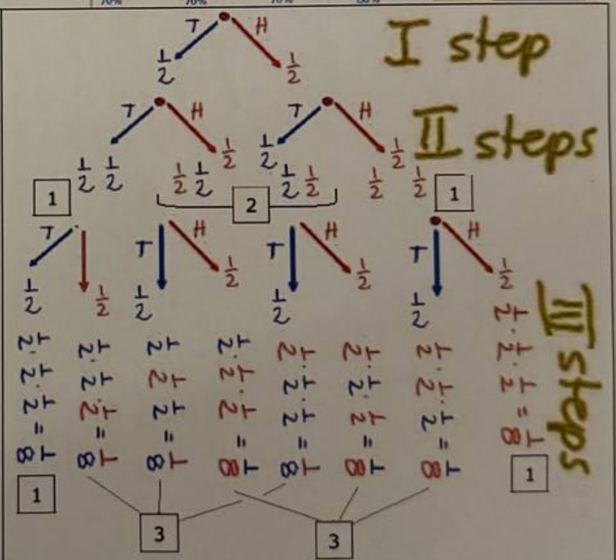
Suppose the events A_j each have probability p, independently

School =formalism=> Uni

1623-1662 Pascal's triangle

the sum of 2 directly above it





$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Newton's Binomial (1642–1727) $(1+x)^p = \cdots$

$\frac{n_1!\, n_2!}{n!}$

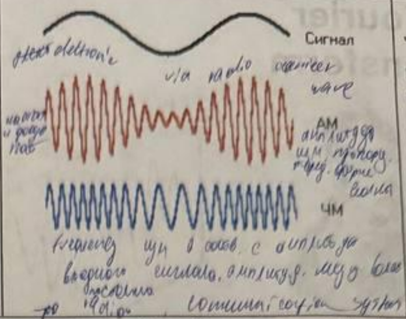
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Pr ABBA

information signal (audio signal, data signal) carries information

Reginald A. Fessenden

(October 6, 1866 – July 22, 1932)

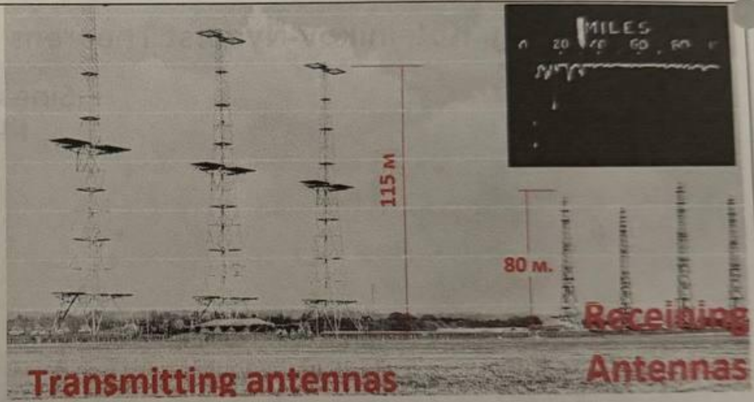


(October 6, 1866 – July 22, 1932)

first transmission of speech by radio (1900), and the first two-way radiotelegraphic communication across the Atlantic Ocean (1906)

"Ни одна организация, занимающаяся какой-либо конкретной областью деятельности, никогда не изобретает какие-либо важные разработки в этой области или не внедряет какие-либо важные разработки в этой области до тех пор, пока она не будет вынуждена сделать это из-за внешней конкуренции.." Oxford University Press. The Quarterly Journal of Economics , Feb., 1926, p. 262.

Battle of Britain
(3 month 3 weeks)
10.07-31.10.1940



Radar played a major role in the Battle of England

early warnings of incoming raids → more effective interception and defense strategies

H. Nyquist

$$W = K \log m$$

Where W is the speed of transmission of intelligence,
m is the number of current values,
and, K is a constant.

Ralph **Hartley**

(81:1888-1970)

$$H = n \log s$$
$$= \log s^n.$$

52! = 225,58 bits

amount of information or uncertainty associated with the arrangement of the cards in a shuffled deck

the entropy for deck is a measure of the uncertainty or disorder in the arrangement of cards math. tool that transforms a signal form its original domain into a representation in the frequency domain

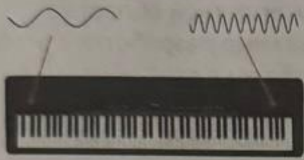
Fourier transform

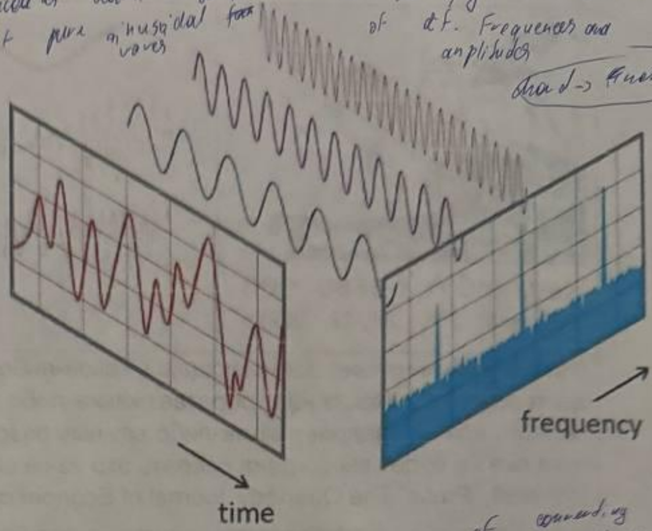
It breaks down complex waveform (sound, img) into collection of pure sinusoidal waves of diff. frequencies and amplitudes

chord -> notes

tells what frequencies are inside the signal

chord -> notes





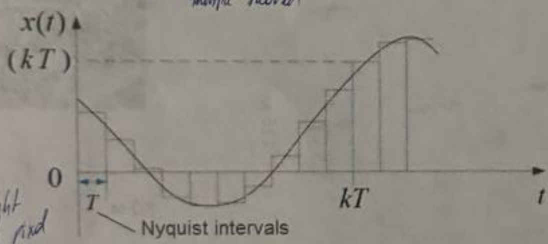
Sampling. Kotelnikov-Nyquist Theorem

simple theorem

process of converting a signal into a sequence of discrete vals (take a snapshot of original at a regular intervals)

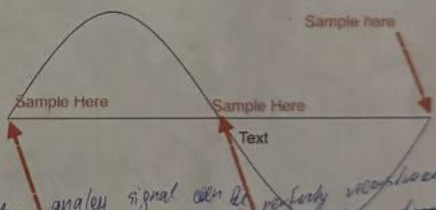
how often do we need to measure smth to capture its shape or behavior

img -> capture light and color on any pixel



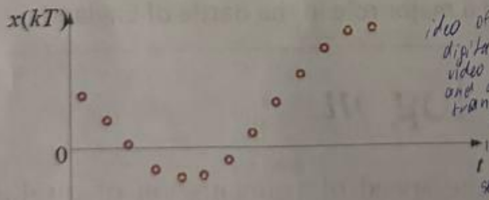
Time intervals T, through which readings s (kT) are taken, are called Nyquist intervals.

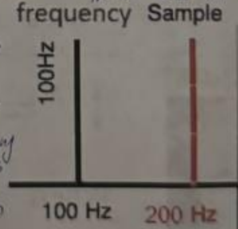
Sine with period T **Sampling at T/2**

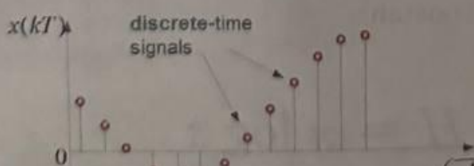


Any analog signal can be perfectly reconstructed from it's digital samples if you sample it at twice its highest frequency

idea of digital audio video and data transmission -> listen song mp3, send voice messages







$$F_{sample} >= 2 * F_{max}$$

$$(T_{sample} <= T_{min}/2)$$

$f=1$, $f=2$, $f=3$ — $\boxed{f\text{-frequency}}$

$T=1$, $T=1/2$, $T=1/3$ $\Big\}$ $T = \frac{1}{f} = f^{-1}$ — $\boxed{T\text{-period}}$

quantity of inf $= \log_2 N$

как перебороть все комбинации в покере, чтобы их не заметить

$B_1 \; B_2$

$2!$

$n! = n(n-1)(n-2)\ldots$

$P(A) = \frac{m_1! \cdot m_2!}{n!}$ $-n-m_1$

$6! \in \underbrace{A_1 A_2 A_3}$ BCD

$3! \in \boxed{B A_2 \; A A_1 \; A_3 C}$

$k=2$

$\frac{n!}{\cancel{m_1! \cdot m_2!}}$

$m_1!(n-m_1)$

$m_2 - f(m_1)$

$m_2 = 2$

$= m_1 + m_2 + m_3$

L M T

$\frac{n!}{m_1! m_2! m_3}$

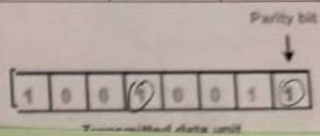
$n = m_1 + m_2 + \ldots m_k$

$\underbrace{X_1 \, X_1}_{m_1}$ (L) $\underbrace{X_1 \, X_2}_{m_2}$ (M) $\underbrace{X_2 \, X_2}_{m_2}$ (T)

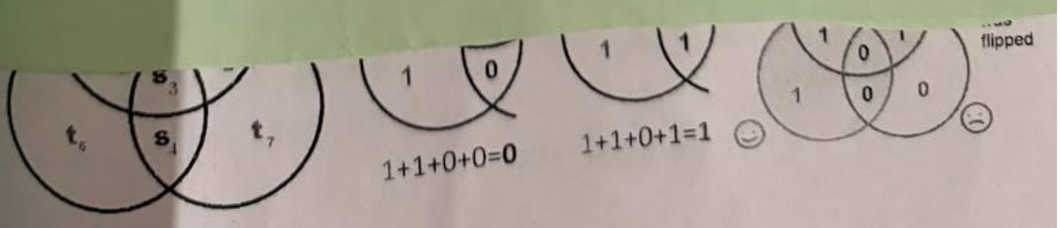
$\frac{n!}{m_1! m_2! \ldots m_k!}$

Sir **Dr. D. MacKay**,
University of Cambridge
(22 April 1967 – 14 April 2016)

"I believe in clean energy,
but I also believe in mathematics"



+0.1 to fin Exam Grade



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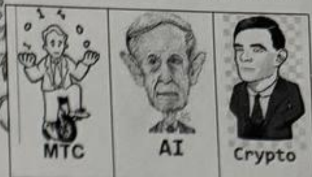
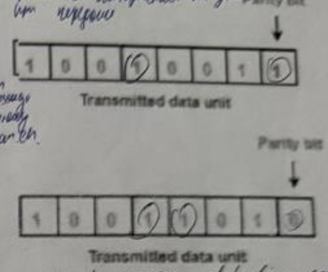
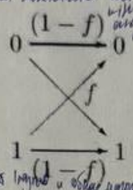


Michael Shellenberger

"I believe in clean energy,
but I also believe in mathematics"



communication theory, focusing on idea of error rates in transmitted messages



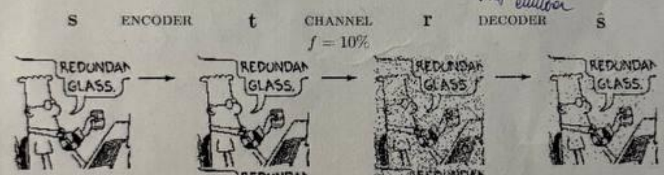
$f = 10\%$

10% вероятность ошибки передачи бита через канал

how errors can occur during transmission

simple error detecting mechanism used in digit. comm. and storage. Extra bit added to string of binary data to help detect whether an error during transmission

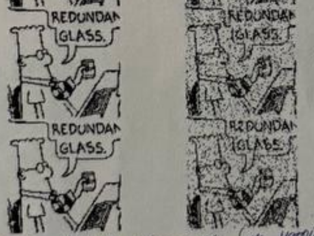
error correction using a repetition code



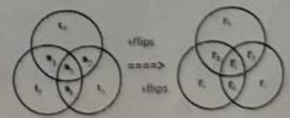
original message to transmit → binary sequence

Source sequence s	Transmitted sequence t
0	000
1	111

The repetition code R_3



source message s	0	0	1	0	1	1	0
t	000	000	111	000	111	111	000
n	000	001	000	000	101	000	000
r	000	001	111	000	010	111	000
corrected errors		*					
undetected errors					*		

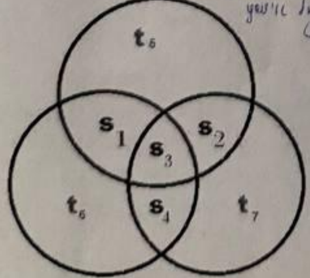


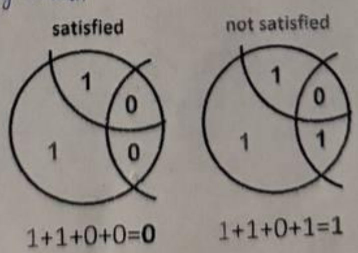
total number of bits in the encoded message

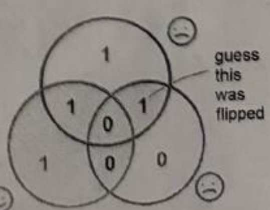
7.4. Hamming code.

$$\frac{4}{\Sigma} \rightarrow \frac{7}{t}$$

data bits (actual information you're trying to send)







7 → 0 111
$P_1 P_2 P_3 D_4$

? ? 0 ? 111

011
$P_1 \rightarrow 0$
$P_2 \rightarrow 0$
$P_4 \rightarrow 1$

0001111

4 ooo ① a Quest

~~[illegible]~~ This 0 } 2 question

Entropy

4 gs

$\log_2 4 = 2$

1: $H(x) = 0{,}5 \cdot 1 + 0{,}3 \cdot 2 + 0{,}1 \cdot 3 + 0{,}1 \cdot 3 = 0{,}5 + 0{,}6 + 0{,}3 + 0{,}3 = 1{,}7$

2: $H(x) = \frac{3}{12} \cdot 1 + \frac{3}{12} \cdot 2 + \frac{3}{12} \cdot 3 + \frac{5}{12} \cdot 3 = 2{,}25$

$$H = \sum_{x=1}^{4} p(x) \log_2\left(\frac{1}{p(x)}\right) \leftarrow \text{Shannon Entropy}$$

$$\boxed{f(x) = \log_2\left(\frac{1}{p(x)}\right)} \leftarrow \text{Quantifying Information}$$

Var. 2:

$H(x) = 0{,}5 \cdot \log_2\left(\frac{1}{0{,}5}\right) + 0{,}3 \cdot \log_2\left(\frac{1}{0{,}3}\right) + 0{,}1 \cdot \log_2\left(\frac{1}{0{,}1}\right) + \ldots$

$= 0{,}5 \cdot \log_2 2 + 0{,}3 \cdot \log_2 3{,}33 + 0{,}1 \log_2 10 + 0{,}1 \cdot \log_2 10 = 0{,}5 \cdot 1 + 0{,}3 \cdot 1{,}74$

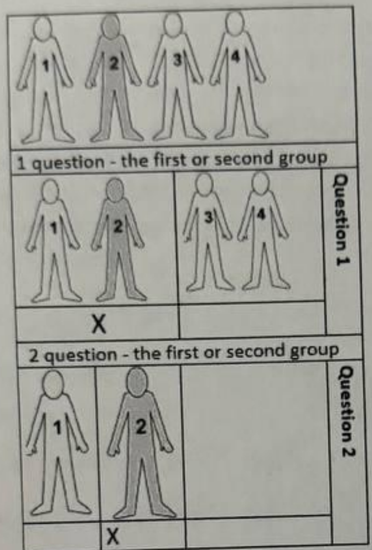
$+ 0{,}1 \cdot 3{,}32 = 1{,}686$

decision-making process

Say **NO** to the **first**

Say **YES** to the **second** if it is better than the first

Say **NO** to the **third** only if it is worse than all the others



y/n questions

how logic and the data affect the average number of steps needed

Average number of questions =

2*0.25+ 2*0.25+ 2*0,25+ 2*0,25 =2

Average number of questions =

1*0.5+	2*0.25+	3*0,125+	3*0,125
			

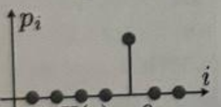
Question 1. Is this Zuckerberg?	 50%	1*0.5
Question 2. Is this Sergey Brin?	 25%	2*0.25
Question 3. Is this Stefan from BMW?	 12,5%	3*0,125
So Prince Saud	 12,5%	3*0,125
Average number of questions =**1,75**		

$$H(X) = \sum_{i=1}^{n} p(x_i) \log_b \frac{1}{p(x_i)}$$

$$\sum_{i=1}^{n} p(i) \log_2 \frac{1}{p(i)}$$

Quantifying information

maximum certainty →



no surprise, no info gained → $H(p) = 0$

partial uncertainty some events more likely than others

$0 < H(p) < \log(n)$

maximum uncertainty all events equally likely →

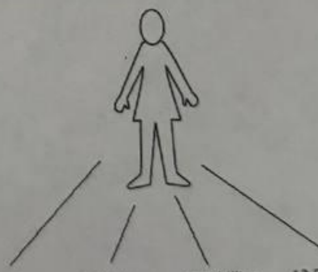
$H(p) = \log(n)$

Shannon's entropy

$$I(x_i) = \log_2\left(\frac{1}{p_i}\right)$$

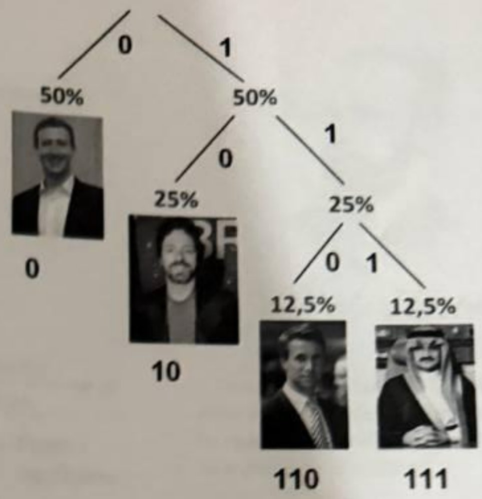
number of bits required to encode choice

$$\sum_{i=1}^{n} p(x_i)\, I(x_i)$$



50 billion	25 billion	12.5 billion	12.5 billion

Mark Zukerberg	Sergey Brin	Stefan Quandt	Prince Al Saud
P(1)= 50%	**P(2)= 25%**	**P(3)= 12,5%**	**P(4)= 12,5%**



First-order approximation (symbols independent but with frequencies of Belarusian txt).

Мама мыла ра

М - 3	30%	1-3 М
а - 4	40%	4-7 а
ы -1	10%	8 -ы
л - 1	10%	9 -л
р - 1	10%	10 -р
10		

лл а $\boxed{\text{мам ма}}$ р

Мама мыла ра

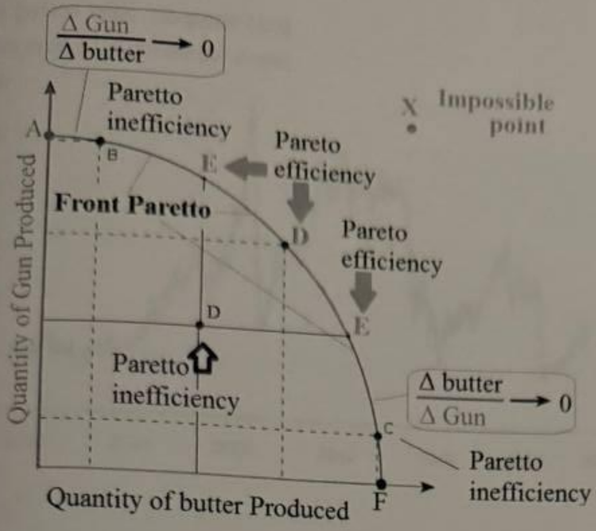
Ма - 2	22%	1-2	ма
ам - 2	22%	3-4	ам
мы - 1	11%	5	мы
ыл - 1	11%	6	ыл
ла - 1	11%	7	ла
а р - 1	11%	8	ар
ра - 1	11%	9	ра
9			

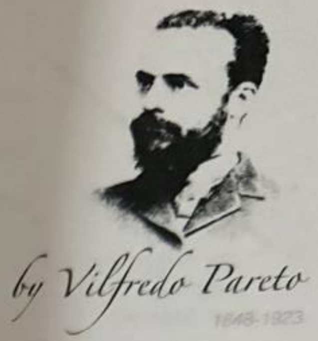
0. 4 6 7 3 1 9 1 6 7 3 5

ам ыл ла ам ма ра ма ыл ла ам мы

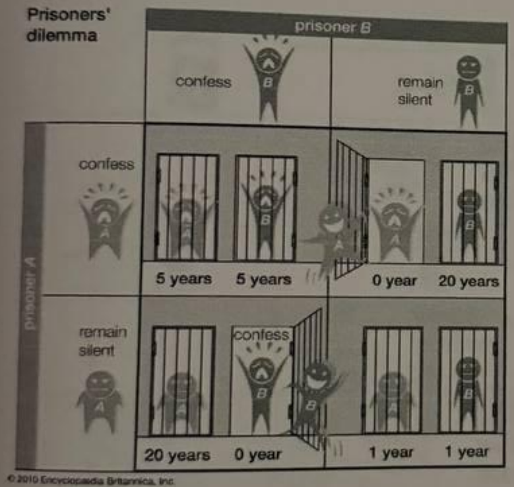
мыллa рама

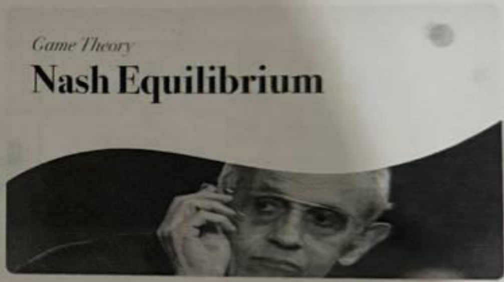
Second-order approximation (digram (2-symbols) structure as in Belarusian)





The orange sector E-D-E is the most Pareto efficient - since an increase in one indicator leads to a decrease in another.

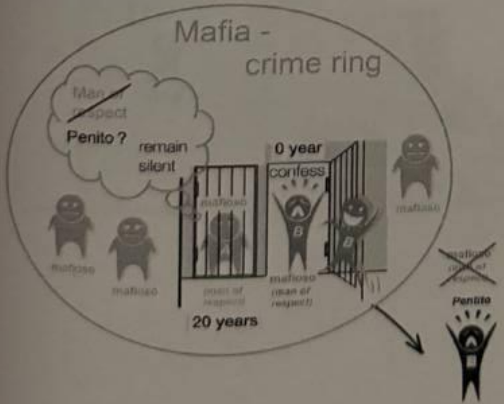




** => Nash equilibrium

$H_2(x)$ / $H_1(x)$	*Player 2*	Recognition;	Non-recognition;
Player 1		1	2
Recognition;	1	* -5 / -5 *	-20 / 0
Non-recognition;	2	0 / -20	-1 / -1

-1-1
Pareto Optimality



Events

consider our "experiment", which has a set of Ω of outcomes $w \in \Omega$

Examples:

1) tossing a coin $\Omega = \{H, T\}$

2) throwing a dice

$\Omega = \{(i,j), i, j \in [1, 2, 3, 4, 5, 6]\}$

A-subset of Ω-event

coin comes up tail $A = \{T\}$ -event

$A = \{(3,6)\}$-event

If $w \in \Omega$ is the outcome, we say A occur if $w \in A$

Complement of A: $\bar{A}$ or A^c -occur when A doesn't occur

difference: $A \setminus B = A \cap B^c$

Intersection: $A \cap B$ occur if both A and B occur

Union: $A \cup B$ occur if any A or B occur

Boolean:

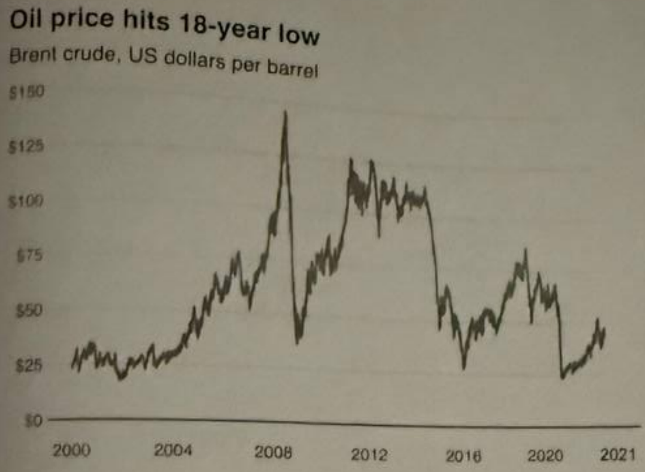
00 | 0
01 | 0
10 | 0
00 | 1

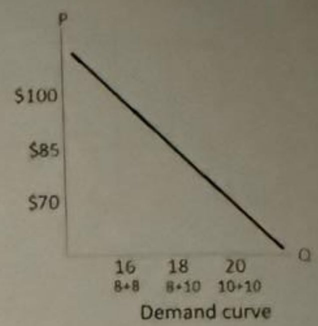
Union: 00 | 0
01 | 1
10 | 1
00 | 1

A and B are disjoint if $A \cap B = \emptyset$

$\Downarrow$

A and B can't occur together





Barrel		1. $8\cdot10^6$ / day	2. $10\cdot10^6$ / day
1.	$8\cdot10^6$	$800 millions per day; $100; $800 millions per day	$850 millions per day; $85; $680
2.	$10\cdot10^6$ / day	$680 millions per day; $85; $850 millions per day	$700 millions per day; $70; $700 millions per day